10-6 Exponential Growth and Decay

Depreciation

$y = a(1 - r)^t$

y - new amt
a - original amt

Decay

$y = ae^{-kt}$

EX: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

$$65 = 130(1 - .11)^t$$
$$\frac{1}{2} = .89^t$$
$$\log\left(\frac{1}{2}\right) = t\log(.89)$$
$$5.9\text{ hrs} = t$$

How long will it take for 90% of this caffeine to be eliminated from a person's body?

$$13 = 130(.89)^t$$
$$\frac{1}{10} = .89^t$$
$$\log\left(\frac{1}{10}\right) = t\log(.89)$$
$$19.8\text{ hrs} = t$$

EX: The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years; that is, every 5760 years, half of a mass of Carbon-14 decays away.

rate

$y = ae^{-kt}$

What is the constant for Carbon-14?

k

$$y = ae^{-kt}$$
$$\frac{1}{2} = 1e^{-k(5760)}$$
$$\frac{1}{2} = e^{-5760k}$$
$$\ln\frac{1}{2} = -5760k$$
$$.00012 = k$$

A paleontologist examining the bones of a wooly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

$$3 = 100 \cdot e^{-.00012t}$$
$$.03 = e^{-.00012t}$$
$$\ln .03 = -.00012t$$
$$29{,}221\text{ yrs} = t$$

EX: The half-life of Sodium-22 is 2.6 years. What is the constant for Sodium-22?

$$\frac{1}{2} = 1e^{-k(2.6)}$$
$$\ln\frac{1}{2} = -2.6k$$
$$.2666 = k$$

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

$.1 = 1e^{-.2666t}$

8.6 yrs

Appreciation

$y = a\ (1 + r\ )^t$

Growth

$y = ae^{kt}$

EX: In 1910 the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in the year 2010?

$y = 120000(1.015)^{100}$

531,845

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